Correspondence

Quasi-TE_{11} Modes in an Anisotropic Plasma Waveguide

The purpose of this correspondence is to demonstrate a fairly simple technique for measuring the propagation constants of the quasi-TE_{11} modes in a cylindrical waveguide partially filled with an anisotropic plasma. A cross section of the system that was investigated is shown in the inset of Fig. 1.

It is well known that, at a given excitation frequency ω, the usual steady-state linearly polarized TE_{11} mode in an empty cylindrical waveguide separates into two quasi-TE modes with different propagation constants and counter-rotating field configurations when an anisotropic plasma column is introduced coaxial with the waveguide axis [1]–[4]. By placing two shorting planes normal to the system axis, a resonance of these modes will occur whenever the separation of the planes L is

$$\beta^{\pm}L = m\pi$$

where $\beta^{\pm}$ are the propagation constants of the quasi-TE_{11}^{+} (the mode which rotates with a right-handed sense with respect to the direction of the DC magnetic field) and quasi-TE_{11}^{-} (the mode which rotates with a left-handed sense) modes, respectively, and m is an integer, excluding zero. We shall label these resonances TE_{11m}^{+} and TE_{11m}^{-}. Note that the distance l between adjacent nodes of the resulting standing wave pattern is

$$l = \pi/\beta^{\pm}. \tag{1}$$

It is possible to measure $\beta^{\pm}$ as a function of frequency by constructing a resonant system and then causing it to resonate in the TE_{11m}^{+} or TE_{11m}^{-} mode at a large number of frequencies by varying L and m. At each resonance, $\beta^{\pm}$ can be found from (1) by measuring l with a moveable detection probe. The system that we have used to make these measurements is shown in Fig. 2. The plasma is the positive column of mercury vapor hot-cathode discharge, the vapor pressure of the mercury corresponding roughly to room temperature. Two metal walls perpendicular to the waveguide axis, which act as shorting planes, complete the resonant system. One wall is movable, so that the distance between walls L is variable [5].

This method of measuring $\beta^{\pm}$ has proved to be useful and has allowed the desired variation of $\beta^{\pm}$ with frequency to be found for the quasi-TE_{11}^{+} and quasi-TE_{11}^{-} modes. This variation is shown in Fig. 1.

Generally, the Q of the resonant system was low, of the order of 50. Nevertheless, the power ratio between successive maxima and minima as measured by the detection probe was large, typically 20 to 30 dB, so that precise measurement of the distance between nodes was always possible. It was found that sufficient points to determine the desired curves could be obtained by varying the length of the resonant system from a maximum of 6.5 inches to about 4 inches, with m taking on the values 2, 3, 4, 5. In Fig. 1 each point is labeled with the appropriate m. The points for which m is circled were obtained with the resonant system at its maximum length. It will be noted that the circled-m points which have the same m do not have exactly the same β. This is probably because

Manuscript received January 26, 1966; revised April 15, 1966. This work was supported under National Science Foundation Grant GK-203, and also under Tri-Service Grants AF-AFOSR 62-340 and AF-AFOSR 139-63.

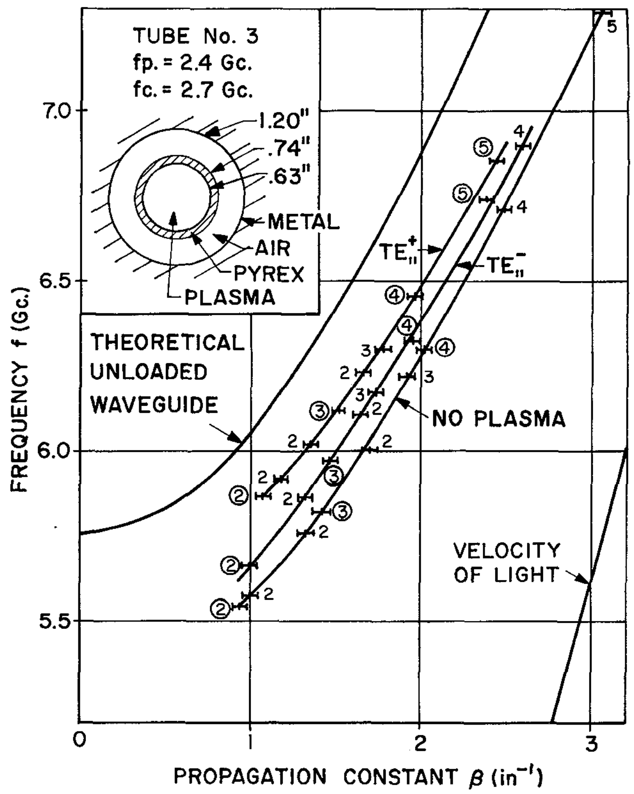

Fig. 1. TE_{11}^{+} and TE_{11}^{-} perturbed waveguide mode Brillouin diagrams. The number beside a point indicates the type of resonance used to obtain the point, i.e., 2 means TE_{112} resonance. A circled number indicates that the cavity was at its maximum length. Dc magnetic field into page in inset.

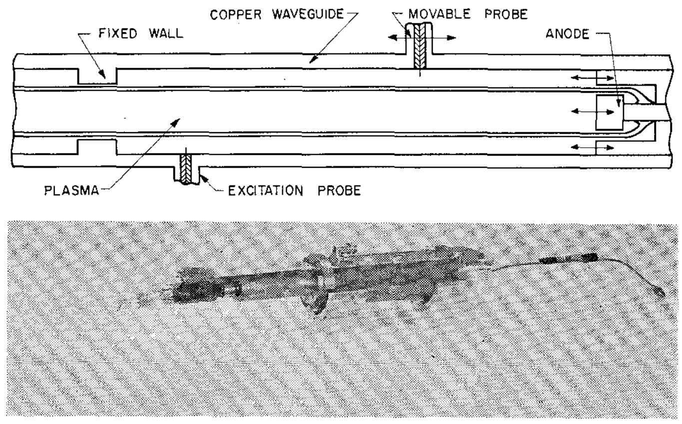

Fig. 2. The resonant system.

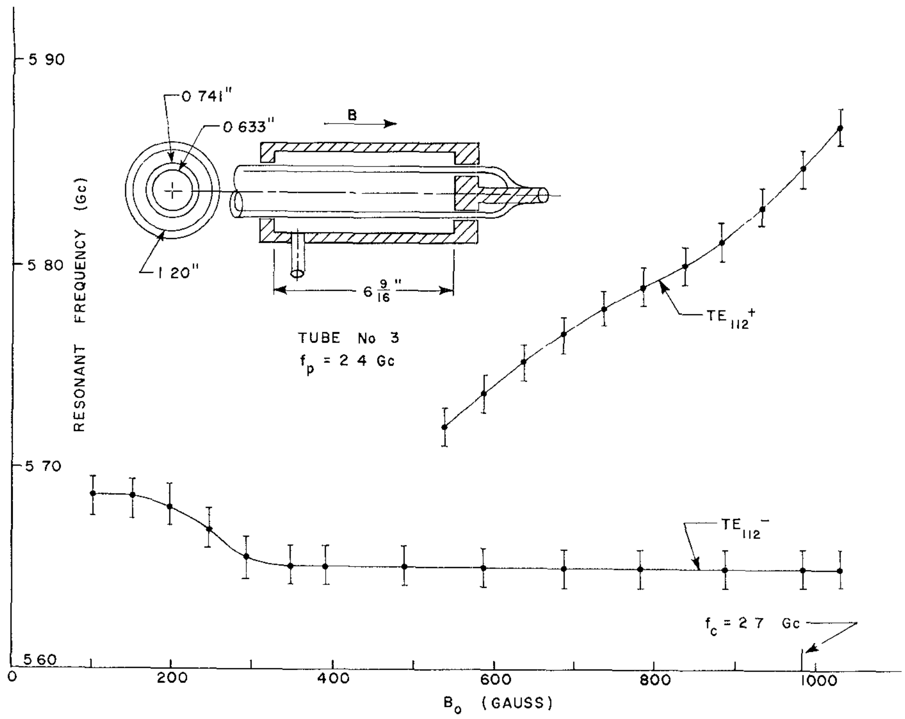

Fig. 3. Variation of resonant frequency of TE_{112}^+ and TE_{112}^- resonances with magnetic field. $f_p = 2.4$ Gc/s.

the "shorting-plane" at the cathode-end of the system is merely a metal ring filling the gap between the glass tube and the copper waveguide, so that the effective position of the reflecting plane at this end may vary slightly with frequency.

The plasma frequency f_p of 2.4 Gc/s indicated in Fig. 1 was determined by the well-known cavity perturbation technique [6]. This measurement required an auxiliary cavity resonant in the TM_{010} mode into which the discharge tube could be inserted coaxially [5].

Figure 3 shows the dependence of the resonant frequencies of the TE_{112}^+ and TE_{112}^- modes on the magnetic field with the resonant system at its maximum length. Careful checking shows that any TE_{11m}^+ resonance will have the same type of dependence on magnetic field as the TE_{112}^+ resonance shown in Fig. 3. Any TE_{11m}^- resonance will have the same type of dependence as the TE_{112}^- resonance.

It is seen that the TE_{112}^- resonance is insensitive to magnetic field except at low fields, while the TE_{112}^+ resonance is quite sensitive to magnetic field. Below 500 gauss it was not possible to detect the TE_{112}^+ resonance because it was masked by the TE_{112}^-, a stronger resonance. Above 1050 gauss it was again not possible to measure the TE_{112}^+ because here it was masked by the TE_{113}^- resonance. These results are in qualitative agreement with the work of Bevc and Everhart [4].

In addition to the signal power appearing at the output of the detection probe, there was also a comparable amount of noise power. The noise power was always nearly proportional to signal power. The presence of noise power did not lessen the accuracy of the measurements because the noise power was a minimum when the signal power was a minimum.

It has been the purpose of this correspondence to demonstrate that the propagation constants of the quasi-TE_{11}^+ and quasi-TE_{11}^- modes may be found with good accuracy by measuring positions of field nodes of resonances derived from these modes. We have also shown of course, that the TE_{11m}^+ and TE_{11m}^- resonances do exist and can be excited easily, at least when the plasma is the positive column of a mercury vapor discharge. While considerable noise power is present, it will not seriously affect the accuracy of the measurements.

R. N. Carlile
Dept. of Elec. Engrg.
University of Arizona
Tucson, Ariz.

References

[1] H. Suhl and L. R. Walker, "Topics in guided wave propagation through gyromagnetic media—part 1," *Bell Sys. Tech. J.*, vol. 33, pp. 579–659, May 1954.
[2] M. Van Trier, "Guided electromagnetic waves in anisotropic media," *Appl. Sci. Res.*, sec. B, vol. 3, pp. 305–371, 1953.
[3] L. Goldstein, "Nonreciprocal electromagnetic wave propagation in ionized gaseous media," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-6, pp. 19–29, January 1958. Also, J. Etter and L. Goldstein, "Guided wave propagation in gyromagnetic gaseous discharge plasma," Univ. of Illinois, Urbana, Elec. Eng. Res. Lab., TR No. 3, October 1954.
[4] V. Bevc and T. E. Everhart, "Fast waves in plasma-filled waveguides," Univ. of California, Berkeley, Inst. of Eng. Res. Tech. Rept., July 1961.
[5] For details, see R. N. Carlile, "Experimental investigation of microwave propagation in a plasma waveguide," Univ. of California, Berkeley, Tech. Mem. No. TM-30, Electronics Research Lab., August 1963.
[6] S. J. Buchsbaum, L. Mower, and S. C. Brown, "Interaction between cold plasma and guided electromagnetic waves," *Phys. Fluids*, vol. 3, pp. 1–14, September–October 1960.

Design of Comb-Line Band-Pass Filters

Comb-line band-pass filters are filters using direct-coupled quarter-wave TEM resonators with adjacent resonators having the *same* open- and short-circuit reference planes. This is in contrast to interdigital band-pass filters where adjacent resonators *alternate* the open- and short-circuit reference planes. Comb-line band-pass filters can be realized using either strip (rectangular center conductors) or *slab* (round center conductors) transmission line resonators. When partitions are employed between adjacent resonators, the comb-line filter structure evolves into a coaxial filter structure. Consequently, comb-line and coaxial nomenclature will be used interchangeably herein. In this correspondence, the published design procedure for comb-line band-pass filters will be related to existing narrow-band filter theory. Certain aspects of the evolution of comb-line structures into coaxial structures will also be discussed.

A procedure has been presented applicable to the design of comb-line band-pass filters of narrow or moderate bandwidth [1]. In this filter structure, use of coupled quarter-wave lines of uniform cross section has been shown to result in an all-stop network. Adjacent resonators are decoupled due to the cancellation of equal magnetic and electric fields in phase opposition. To achieve band-pass filter behavior, resonator lines are foreshortened to electrical lengths less than ninety degrees, and resonance is obtained using lumped-capacitances at the resonator open-ends. Appreciable coupling between adjacent resonators is realized because the magnetic and electric couplings are no longer equal. In this case, the net coupling will be magnetic.

For narrow-band filters (i.e., bandwidth less than ten percent), it is convenient to describe the interstage couplings by coefficients of coupling [2] which can be readily measured by simple experimental procedures [3]. Assuming all interstage couplings to be equal, Matthaei's design equations [1] can be rewritten as follows, when $Y_{AK} = Y_A$:

$$\frac{C_{ij}}{\epsilon} = \frac{377\,Y_A}{\sqrt{\epsilon_r}}\left(\frac{J_{ij}}{Y_A}\right)\tan\theta_0 \qquad (1)$$

$$\frac{J_{ij}}{Y_A} = \frac{w}{\omega_1{}'}\left(\frac{b}{Y_A}\right)\frac{1}{\sqrt{g_i g_j}} \qquad (2)$$

$$\frac{b}{Y_A} = \left(\frac{\cot\theta_0 + \theta_0\csc^2\theta_0}{2}\right). \qquad (3)$$

Letting $w = 1/Q_T$, $\omega_1{}' = 1.0$, and

$$K_{ij} = \frac{1}{Q_T}\sqrt{\frac{1}{g_i g_j}},$$

and combining (1), (2), and (3), it can be shown that

$$\frac{C_{ij}}{\epsilon} = \frac{377\,Y_A}{\sqrt{\epsilon_r}}K_{ij}f(\theta_0) \qquad (4)$$

Manuscript received April 4, 1966.